

Neutrino masses and mixings in a seesaw framework

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Assuming the seesaw mechanism for hierarchical neutrino masses, we calculate the heavy neutrino masses under the hypotheses that the mixing in the Dirac leptonic sector is similar to the quark mixing ($V_D \sim V_{CKM}$) and that $M_\nu \sim M_u$ or M_e , where M_ν is the Dirac mass matrix of neutrinos. As a result we find that for $M_\nu \sim M_u$ the vacuum oscillation solution of the solar neutrino problem leads to a scale for the heavy neutrino mass well above the unification scale, while for the MSW solutions there is agreement with this scale. For $M_\nu \sim M_e$ the vacuum solution is consistent with the unification scale, and the MSW solutions with an intermediate scale. The mass of the lightest heavy neutrino can be as small as 10^5 GeV.

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I. INTRODUCTION

In the Minimal Standard Model (MSM) [1] the neutrino is massless. This is because with only the left-handed neutrino ν_L we cannot build a Dirac mass term, and with only the Higgs doublet ϕ we cannot build a Majorana mass term for ν_L after spontaneous symmetry breaking. However, strong indications for nonzero neutrino mass come from solar and atmospheric neutrino experiments and on the theoretical side there are several extensions of the MSM that lead to nonzero neutrino mass [2].

The simplest one is to add the right-handed neutrino ν_R in order to have the analogous of the quark u_R in the leptonic sector. When this is done, it becomes possible to give a Dirac mass to the neutrino by means of the same mechanism used for the other fermions. Thus we expect this mass to be of the same order of magnitude of the other fermion masses. Moreover, it is now also possible to have a bare Majorana mass term for ν_R , and the corresponding value of the mass is not constrained if the gauge group is the same of the MSM. Therefore we have a new mass scale in the extended theory [3], and it is a key problem to understand if this new scale is associated to new physics, that is a larger gauge group, and at what energy it eventually happens.

If the Dirac mass of the neutrino is of the same order of the other quark or lepton masses, the seesaw mechanism [4] relates the smallness of the neutrino mass to a very large scale in the Majorana term. Of course we have three generations of fermions and we expect three light neutrinos and three heavy ones. We assume that the light neutrino mass spectrum is hierarchical as it happens for the quark and charged lepton mass spectra. We denote by m_1, m_2, m_3 the Dirac masses, by M_1, M_2, M_3 the heavy neutrino masses, and by $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ the light neutrino masses. From the solar and atmospheric neutrino experiments we can infer [5], with some uncertainty, the values of m_{ν_2} and m_{ν_3} , and of the neutrino mixing matrix U ,

$$\nu_\alpha = U_{\alpha i} \nu_i, \tag{1}$$

where U is unitary and connects the mass eigenstates ν_i ($i = 1, 2, 3$) to the weak eigenstates ν_α ($\alpha = e, \mu, \tau$).

The aim of this paper is to calculate the heavy neutrino masses under simple hypotheses on the Dirac masses of neutrinos and on the matrix

$$V_D = V_\nu^\dagger V_e \quad (2)$$

which is the analogous of the CKM matrix in the Dirac leptonic sector. Namely, we assume $V_D \sim V_{CKM}$, and $M_\nu \sim M_u$ or $M_\nu \sim M_e$. For m_{ν_1} we allow a variation of three orders in the hierarchical regime. We are mostly interested in M_3 (the mass of the heaviest right-handed neutrino), which is related to the new mass scale of the theory, and in M_1 (the mass of the lightest right-handed neutrino), which has some importance in baryogenesis *via* leptogenesis [6,7]. In Grand Unified Theories (GUTs) M_3 is associated to unification or intermediate scales [8], thus we match our results with these scales. General considerations on heavy neutrino masses in the seesaw mechanism can be found in ref. [9]. In the present paper we give a numerical analysis based on the hypotheses above and the experimental data on solar and atmospheric neutrinos.

II. THEORY

Let us briefly explain the effect of the seesaw mechanism on the leptonic mixing. The part of the Lagrangian we have to consider is

$$\bar{e}_L M_e e_R + \bar{\nu}_L M_\nu \nu_R + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M'_R \nu_R \quad (3)$$

where M_e and M_ν are the Dirac mass matrices of charged leptons and neutrinos respectively, and M'_R is the Majorana mass matrix of right-handed neutrinos. If we assume the elements of M'_R much greater than those of M_ν , the seesaw mechanism leads to the effective Lagrangian

$$\bar{e}_L M_e e_R + \bar{\nu}_L M'_L \nu_R^c + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M'_R \nu_R \quad (4)$$

where

$$M'_L = M_\nu M'^{-1}_R M_\nu^T \quad (5)$$

is the Majorana mass matrix of left-handed neutrinos (in this context the left-handed neutrinos are called light neutrinos and the right-handed neutrinos are called heavy neutrinos). Diagonalization of M_e , M'_L gives (renaming the fermion fields)

$$\bar{e}_L D_e e_R + \bar{\nu}_L D_L \nu_R^c + g \bar{\nu}_L V_{lep} e_L W + \bar{\nu}_L^c M'_R \nu_R. \quad (6)$$

Of course we can also diagonalize M'_R without changing other parts of this Lagrangian. The unitary matrix V_{lep} [10] describes the weak interactions of light neutrinos with charged leptons. The following steps clarify the structure of V_{lep} .

If in eqn.(3) we first diagonalize M_e and M_ν , obtaining

$$\bar{e}_L D_e e_R + \bar{\nu}_L D_\nu \nu_R + g \bar{\nu}_L V_D e_L W + \bar{\nu}_L^c M_R \nu_R, \quad (7)$$

then the seesaw mechanism gives

$$\bar{e}_L D_e e_R + \bar{\nu}_L M_L \nu_R^c + g \bar{\nu}_L V_D e_L W + \bar{\nu}_L^c M_R \nu_R \quad (8)$$

with

$$M_L = D_\nu M_R^{-1} D_\nu. \quad (9)$$

Then, we diagonalize also M_L ,

$$\bar{e}_L D_e e_R + \bar{\nu}_L D_L \nu_R^c + g \bar{\nu}_L V_s V_D e_L W + \bar{\nu}_L^c M_R \nu_R, \quad (10)$$

and, comparing with eqn.(6), we recognize that

$$V_{lep} = V_s V_D \quad (11)$$

where

$$V_s M_L V_s^T = D_L. \quad (12)$$

We also understand that

$$V_{lep} = U^\dagger, \quad (13)$$

and point out that M'_R (M'_L) differs from M_R (M_L) by a unitary transformation, hence they have the same eigenvalues.

In the Lagrangian (3) it is possible to diagonalize M_e or M_R without changing the observables quantities. The same is not true for M_ν . Moreover, M_e and M_R can be diagonalized simultaneously. In the Lagrangian (4) the following matrices can be diagonalized: M_e , M_L , M_R , both M_L and M_R , both M_e and M_R . When we set $M_e = D_e$ in eqn.(4) we have $M_L = U D_L U^T$, and when we set $M_L = D_L$ we get $M_e = U^\dagger D_e U$ if M_e is chosen hermitian or $M_e M_e^\dagger = U^\dagger D_e^2 U$ if M_e contains three zeros [11].

III. EXPERIMENT

Experimental informations on neutrino masses and mixings are increasing rapidly. To be definite we refer to [5], where the matrix U is written as

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}. \quad (14)$$

There is a zero in position 1-3, although it is only constrained to be much less than one [12]. The experimental data on oscillation of atmospheric and solar neutrinos lead to three possible numerical forms for U , corresponding to the three solutions of the solar neutrino problem, namely small mixing and large mixing MSW (smMSW and lmMSW) [13], and vacuum oscillations (VO) [10,14,15]. Choosing the central values of neutrino masses and of s_{12} and s_{23} from ref. [5], we have always $m_{\nu_3} = 5.7 \times 10^{-11}$ GeV, and

$$U = \begin{pmatrix} 1 & 0.04 & 0 \\ -0.032 & 0.80 & 0.60 \\ 0.024 & -0.60 & 0.80 \end{pmatrix} \equiv U_1, \quad (15)$$

$m_{\nu_2} = 2.8 \times 10^{-12}$ GeV for small mixing MSW,

$$U = \begin{pmatrix} 0.91 & 0.42 & 0 \\ -0.336 & 0.726 & 0.60 \\ 0.252 & -0.544 & 0.80 \end{pmatrix} \equiv U_2, \quad (16)$$

$m_{\nu_2} = 4.4 \times 10^{-12}$ GeV for large mixing MSW, and

$$U = \begin{pmatrix} 0.80 & 0.60 & 0 \\ -0.474 & 0.632 & 0.61 \\ 0.366 & -0.488 & 0.79 \end{pmatrix} \equiv U_3, \quad (17)$$

$m_{\nu_2} = 9.2 \times 10^{-15}$ GeV for vacuum oscillations. We also consider maximal and bimaximal [16] mixing as limiting cases of U_1 and U_3 , respectively:

$$U_m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (18)$$

$$U_b = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (19)$$

Experimental data on oscillations only give Δm_{32}^2 , Δm_{21}^2 , from which, for hierarchical light neutrino masses, we yield the values of m_{ν_3} , m_{ν_2} , because $\Delta m_{32}^2 \simeq m_{\nu_3}^2$, $\Delta m_{21}^2 \simeq m_{\nu_2}^2$. For m_{ν_1} we will assume $m_{\nu_1} \leq 10^{-1} m_{\nu_2}$. From the unitary matrices written above we see that leptonic mixing between second and third family is large, while the mixing between first and second family may be large or small. It is well-known that in the quark sector all mixings are small.

IV. CALCULATION

Our determination of M_1 , M_2 , M_3 is based on the following assumptions. Looking at eqn.(11) we see that V_s could be responsible of the enhancement of lepton mixing [17]. Therefore, it is suggestive to assume that the matrix V_D has just the form of the CKM matrix

$$V_D = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & \lambda^4 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & \lambda^2 \\ \lambda^3 - \lambda^4 & -\lambda^2 & 1 \end{pmatrix}. \quad (20)$$

This is similar to the form which might originate from GUTs [18]

$$V_D = \begin{pmatrix} 1 - \frac{1}{18}\lambda^2 & \frac{1}{3}\lambda & \lambda^4 \\ -\frac{1}{3}\lambda & 1 - \frac{1}{18}\lambda^2 & \lambda^2 \\ \frac{1}{3}\lambda^3 - \lambda^4 & -\lambda^2 & 1 \end{pmatrix}, \quad (21)$$

the difference being in the element V_{D12} , and this, in turn, to the V_D which results from the analogy of ref. [19], where $V_{D23} \simeq 2\lambda^2$. We also assume $M_\nu \sim M_u$ or $M_\nu \sim M_e$. In particular,

$$M_\nu = D_\nu = \frac{m_\tau}{m_b} D_u, \quad (22)$$

where the factor is due to running [20], or

$$M_\nu = D_\nu = D_e. \quad (23)$$

We use quark (and charged lepton) masses at the scale M_Z as in ref. [21]. It is important to notice that the values M_1, M_2, M_3 do not depend on the assumption $M_\nu = D_\nu$, because M_R undergoes a unitary transformation. Also V_D does not change if we rotate e_L as ν_L . In fact, one can always diagonalize M_u without changing V_{CKM} [22], and M_ν without changing both V_D and M_1, M_2, M_3 . We vary m_{ν_1} by three orders down from $m_{\nu_1} = 10^{-1}m_{\nu_2}$ to $m_{\nu_1} = 10^{-4}m_{\nu_2}$. In the tables we report

our results (notation: $xy \equiv x \times 10^y$, in GeV). They are obtained in the following way. From eqn.(11) we have

$$V_s = V_{lep} V_D^\dagger; \quad (24)$$

using eqn.(12) we get

$$M_L = V_s^T D_L V_s, \quad (25)$$

and from eqn.(9) we obtain

$$M_R = D_\nu M_L^{-1} D_\nu \quad (26)$$

and then its eigenvalues. We see that in the case $M_\nu \sim M_u$ the VO solution leads to a scale for M_3 well above the unification scale (around the Planck scale), while the MSW solutions are consistent with this scale. M_2 is around the intermediate scale. In the case $M_\nu \sim M_e$ the VO solution gives M_3 near the unification scale, while the MSW solutions bring it near an intermediate scale. Also we notice that M_1 may be of the order 10^6 , a relatively small value [19,6]. The huge value of M_3 in the VO case is due mainly to the lower values of m_{ν_1} , m_{ν_2} respect to the MSW case. There is no substantial change if the factor m_τ/m_b is erased from eqn.(22): the numerical results are rescaled by the value 2.8. We have introduced such a factor because the relation $M_u \sim M_\nu$ is typical of GUTs, where it is true at the unification scale, while the factor m_τ/m_b appears at low energy due to running. It can be checked that there is not an essential difference between the results obtained by eqn.(20) ($V_D \sim V_{CKM}$) and those obtained by eqn.(21) ($V_D \sim V_{GUT}$). In fact, numbers differ by no more than one order of magnitude. Moreover, comparing values in the two MSW cases, the effect of changes in m_{ν_1} is apparent: in the small mixing solution M_3 varies by one order, in the large mixing solution by three orders. If we want M_3 not to exceed the unification scale, then m_{ν_1} cannot be much smaller than m_{ν_2} , in the large mixing MSW. Maximal and bimaximal mixings confirm the results obtained for small mixing MSW and vacuum oscillations, respectively.

	smMSW	lmMSW	VO	max	bimax
M_1	1.3e6	6.8e5	2.5e6	1.1e6	4.0e6
	1.6e6	2.3e6	3.1e9	1.2e6	3.7e9
M_2	3.1e10	1.5e10	1.7e12	3.0e10	1.4e12
	9.4e11	2.0e10	2.0e12	1.1e13	1.5e12
M_3	1.7e15	2.8e15	2.2e18	2.2e15	3.7e18
	4.7e16	1.9e18	1.9e21	5.0e15	3.3e21

Table 1: $V_D \sim V_{CKM}, M_\nu \sim M_u$

	smMSW	lmMSW	VO	max	bimax
M_1	1.7e5	8.6e4	2.2e5	1.4e5	1.7e5
	2.0e5	9.7e4	1.3e7	1.6e5	1.4e7
M_2	2.1e9	1.0e9	1.2e11	2.0e9	9.3e10
	5.4e10	1.3e9	1.4e11	3.8e11	1.0e11
M_3	5.0e11	7.9e11	6.1e14	6.2e11	1.0e15
	1.5e13	5.3e14	5.2e17	2.8e12	9.3e17

Table 2: $V_D \sim V_{CKM}, M_\nu \sim M_e$

As a matter of fact $V_D \sim V_{CKM}$ and $V_D \sim V_{GUT}$ are not so different from $V_D \sim I$. In such a case $V_{lep} \simeq V_s$. The opposite case is $V_{lep} \simeq V_D$ and then $V_s \simeq I$, that is, when $M_\nu = D_\nu$ also $M_R = D_R$. From the seesaw mechanism we obtain

$$M_i = \frac{m_i^2}{m_{\nu_i}}, \quad (27)$$

which gives $M_3 \sim 10^{14}$, $M_2 \sim 10^{10}$, $M_1 \sim 10^6 \div 10^9$ (MSW), $M_2 \sim 10^{13}$, $M_1 \sim 10^9 \div 10^{12}$ (VO) GeV in the case $M_\nu \sim M_u$; $M_3 \sim 10^{10}$, $M_2 \sim 10^9$, $M_1 \sim 10^5 \div 10^8$ (MSW), $M_2 \sim 10^{12}$, $M_1 \sim 10^8 \div 10^{11}$ (VO) GeV in the case $M_\nu \sim M_e$. In the VO solution with $M_\nu \sim M_e$, M_2 exceeds M_3 , and also M_1 can do it.

Let us now briefly discuss the sensitivity to input mixing angles of the results reported in the first three columns of tables 1,2. By allowing s_{12} and s_{23} to vary inside the ranges reported in ref. [5] we have found that the numerical values of right-handed neutrino masses change by no more than one order of magnitude. The same happens if U_{13} is different from zero up to 0.1. Therefore the above considerations on the physical scales do not change.

It is also interesting to match our results, obtained by a hierarchical spectrum, with the degenerate spectrum and the democratic mixing [23]

$$U_d = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (28)$$

Assuming as light neutrino mass $m_0 = 2$ eV, relevant for hot dark matter, and the democrating mixing $U = U_d$, we obtain $M_1 = 9.2 \times 10^2$, $M_2 = 7.7 \times 10^7$, $M_3 = 5.5 \times 10^{12}$ GeV for $M_\nu \sim M_u$ and $M_1 = 1.2 \times 10^2$, $M_2 = 5.3 \times 10^6$, $M_3 = 1.5 \times 10^9$ GeV for $M_\nu \sim M_e$, that is M_3 at the intermediate scale and M_1 even at the electroweak scale. From eqns.(25),(26) we see that in the case of degenerate masses M_R is proportional to $1/m_0$ and one can easily obtain M_3 when m_0 is lowered.

V. CONCLUSION

We have calculated the heavy neutrino masses in a seesaw framework, under simple hypotheses on the Dirac sector, and using experimental limits on light neutrino masses and mixings. The results have been matched with intermediate and unification scales. A key result is that the large mixing MSW solution can be reconciled with GUTs. The analysis can be improved when more precise data will be available. Also the effect of phases should be considered [24]. There are several recent studies about the seesaw mechanism [25], based on various forms of mass matrices; a nice review is in ref. [26]. Instead, in this paper, we work on the matrix V_D and on D_ν , that is the leptonic quantities which correspond, in the quark sector, to the observable quantities.

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